Journal of Applied Mechanics and Technical Physics, Vol. 47, No. 6, pp. 905–910, 2006

MESOSCALE MEASUREMENT OF STRAINS BY ANALYZING OPTICAL IMAGES OF THE SURFACE OF LOADED SOLIDS

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UDC 004.932

An algorithm for estimating the strains in solids at the mesoscale level, which is based on constructing fields of displacement vectors and calculating strain components, is studied and tested. Verification of the method is performed by comparing the analytically calculated strain components with experimental data obtained on the basis of model images and images recorded by a TOMSC *optical* TV *measurement system during tension of polymer samples. The estimates of the strain components obtained by a correlation analysis of images are shown to be in good agreement with the results of analytical calculations for known loading parameters and sizes of the images under analysis. The results of the study confirm that the* TOMSC *system can be used to estimate local strains by processing images of the surface of loaded samples of materials.*

Key words: *field of displacement vectors, image processing, strain, correlation analysis.*

Introduction. Several methods are currently used to study the strain parameters and for nonintrusive monitoring of materials and structural elements. Each method has its own merits and drawbacks associated with the possibility of using the method under particular conditions, its accuracy, probability of correct identification of finite-sized defects, sensitivity, cost of manufacturing and operation of the instruments, difficulties in measurements, etc.

An optical TV method of constructing the field of displacement vectors and calculating the strain components is considered in the present paper. The objective of the work is to test the method for estimating the strains in solids at the mesoscale level with the use of a series of model and experimental images reflecting the changes in shape of the samples under different types of loading.

1. Method of Strain Estimates Based on the Correlation Analysis of Images. The algorithm of displacement estimates is based on calculating the correlation function and searching for the maximum of this function. We used the following relation for calculating the correlation coefficient:

$$k_r = \sum_{i=1}^{n}\sum_{j=1}^{n} I_{1,i,j} I_{2,i,j} \Big/ \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} I_{2,i,j}^2}.$$

Here I_1 and I_2 are the brightnesses of elements (pixels) of the compared portions of the images and n is the length of the side of the elementary area for which the coefficient k_r is calculated. The maximum value of the coefficient k_r within the scanned zone is found line-by-line with a step of 1 pixel [1–3]. The size of the scanned zone is defined by the parameter r; the vectors are constructed with a prescribed step. The values of the parameters k_r and r and the step for vector construction are defined independent of each other.

After the displacement of the area is determined with accuracy to one pixel, the value of the displacement has to be refined to fractions of a pixel. The subpixel accuracy is reached by bicubic (two-dimensional) interpolation of nodes with the greatest values of k_r, i.e., only one segment rather than the entire distribution of the correlation

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function in the computational domain is interpolated. After interpolation, the maximum of the bicubic spline function is found, whose coordinates determine the displacement magnitude and direction.

Quantitative characteristics of strain are found by numerical differentiation of the components of the field of displacement vectors [2, 3].

2. Modeling of Surface Deformation. The method proposed was verified by test computations with the use of model and experimental images of the surface with known parameters of deformation. The test computations with a series of model images were designed to test the method in the absence of external disturbances, such as noise, geometrical distortions, etc. The following approach was used to generate model images corresponding to different schemes of material loading.

The image of a real surface is an optical image, and each portion (area) of this image is characterized by a certain brightness (intensity of reflected light). In the further description of image modeling, the surface is understood as a continuous distribution of brightness (optical image). The process of constructing model images can be divided into several stages:

1) Obtaining a set of discrete values of brightness (node points) of the surface with a prescribed distribution;

2) Constructing a continuous distribution of brightness (surface) by interpolation of these node points;

3) Setting the strain parameters (type and increment) and recalculating the surface with allowance for the defined values;

4) Discretizing the surface for obtaining a model image.

As the present study is not aimed at identification of characteristic objects in the image, the model of an optical image is a background. The background can be considered as a random process with some features that are not inherent in the object, which complicates identification of information features of the object. It should also be taken into account that real images contain noise and distortions arising during transformation of a two-dimensional brightness signal at the sensor input into electric current at the output, for instance, noise caused by nonuniform sensitivity of the sensor over the field, geometrical distortions, noise of signal discretization, etc. [4].

The model images were similar images with different statistical parameters (mean level of brightness, variance). We can simply and effectively model background images in the form of two-dimensional stochastic fields with prescribed statistical properties by relations of the form [4]

$$F_{x,y} = \alpha F_{x,y-1} + \beta F_{x-1,y} - \alpha\beta F_{x-1,y-1} + n_{x,y}\sigma\sqrt{(1-\alpha)(1-\beta)} + m(1-\alpha)(1-\beta), \tag{1}$$

where $F_{x,y}$ is the current value of the background at the point (x, y), α and β are the coefficients of correlation between the neighboring elements in the horizontal and vertical directions, respectively, $n_{x,y}$ is a random sequence of numbers with a zero mean value and unit variance, σ is the prescribed root-mean-square deviation of the image brightness amplitudes, and m is the prescribed mathematical expectation of the image brightness amplitudes.

By varying the parameters m, σ, α, and β in Eq. (1), we can obtain a wide range of background images with controlled statistical parameters. For fixed statistical parameters, it is possible to create a database of similar, which, in turn, allows more accurate numerical experiments. For $\alpha = 0$ and $\beta = 0$, the background model reduces to a random sequence $n_{x,y}$, which is brought to a needed scale (in terms of image pixel brightness) with the use of the parameters m and σ. A uniform distribution was used as a distribution of the random sequence.

After a two-dimensional set of nodes with prescribed statistical parameters was calculated, the nodes were interpolated to obtain a continuous distribution of brightness. The distance between the nodes was assumed to be 1 pixel. Thus, the entire model was divided into square areas; interpolation was performed by bicubic two-dimensional splines of the form

$$f(x,y) = a_0x^3 + a_1x^2 + a_2x + a_3y^3 + a_4y^2 + a_5y + a_6x^3y^3 + a_7x^3y^2 + a_8x^3y + a_9x^2y^3 + a_{10}xy^3 + a_{11}x^2y^2 + a_{12}x^2y + a_{13}xy^2 + a_{14}.$$

Surface discretization implies finding the brightness of each pixel of the image after deformation. The brightness of each pixel is determined as the mean value of brightness of the surface area corresponding to the position and size of the pixel.

A model with a uniform distribution with the parameters $m = 150$ and $\sigma = 40$ was used in further test computations.

2.1. *Model of Biaxial Tension of the Surface.* We consider a rectangular portion (area) of the surface of size $l \times w$ (Fig. 1a). Under uniform tension of the surface in the x and y directions, the values of l and w increase by

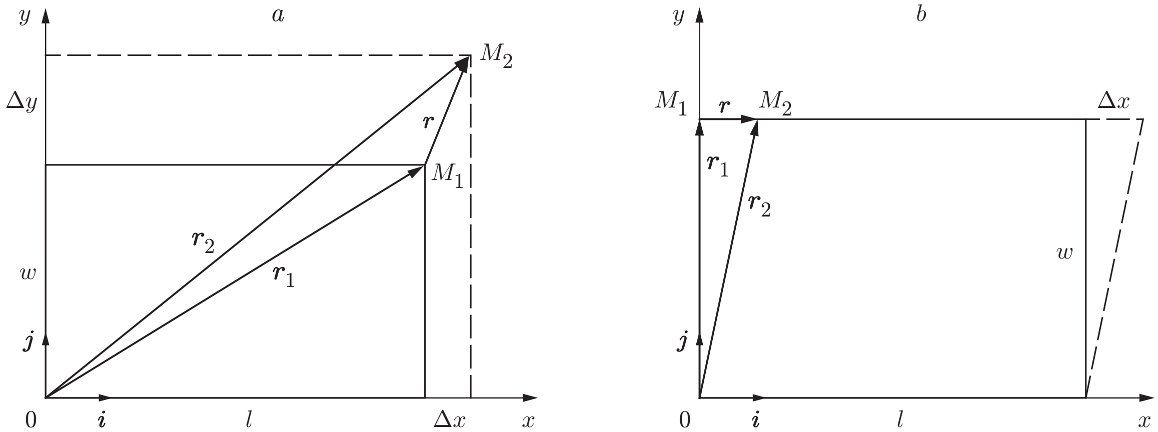

Fig. 1. Deformation of a rectangular area: biaxial tension (a) and uniaxial shear (b).

Δx and Δy, respectively. The origin is located in the left bottom corner of the rectangle. We write the coordinates of the radius-vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$:

$$\boldsymbol{r}_1 = (l, w), \qquad \boldsymbol{r}_2 = (l + \Delta x, w + \Delta y).$$

After deformation, the point M_1 transforms to the point M_2, and the vector $\boldsymbol{r}$ equals the difference between the vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$:

$$\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1 = (l + \Delta x - l, w + \Delta y - w) = (\Delta x, \Delta y).$$

As the tension is uniform, the displacement vector for each point of the surface has the form

$$\boldsymbol{r}_{xy} = (k_1 x, k_2 y), \qquad k_1 = \Delta x / l, \quad k_2 = \Delta y / w$$

(x and y are the point coordinates). For all points of the surface, we can write the vector field in Cartesian coordinates:

$$\boldsymbol{V}(x, y) = V_x(x, y) \cdot \boldsymbol{i} + V_y(x, y) \cdot \boldsymbol{j}, \qquad V_x(x, y) = k_1 x, \quad V_y(x, y) = k_2 y.$$

It is convenient to use the distortion tensor to describe the strains in the object. We give the expressions for the longitudinal (ε_{xx}), transverse (ε_{yy}), and shear (ε_{xy}) components of the distortion tensor [1, 2]:

$$\varepsilon_{xx} = \frac{\partial V_x}{\partial x}, \qquad \varepsilon_{yy} = \frac{\partial V_y}{\partial y}, \qquad \varepsilon_{xy} = \frac{1}{2}\Big(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\Big).$$

We also write the expression for the shear strain intensity γ_i [5, 6]:

$$\gamma_i = \sqrt{2/3}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \varepsilon_{xx}^2 + \varepsilon_{yy}^2 + (3/2)\varepsilon_{xy}^2}. \tag{2}$$

Knowing the field of displacement vectors, we can determine the above-mentioned components of the distortion tensor and the shear strain intensity:

$$\varepsilon_{xx} = \frac{\partial\,(k_1 x)}{\partial x} = k_1, \qquad \varepsilon_{yy} = \frac{\partial\,(k_2 y)}{\partial y} = k_2, \qquad \varepsilon_{xy} = \frac{1}{2}\Big(\frac{\partial\,(k_1 x)}{\partial y} + \frac{\partial\,(k_2 y)}{\partial x}\Big) = 0,$$
$$\gamma_i = \sqrt{2/3}\,\sqrt{(k_1 - k_2)^2 + k_1^2 + k_2^2}. \tag{3}$$

The size of the model images used in the present work was 512×512 pixels. Four series of experiments were performed, with 11 images each; the increment of length l and width w of the model surface at each step was 0.5 pixel. The fields of displacement vectors were constructed with the use of the above-described algorithm with the following values of parameters: size of the area for computing the correlation coefficient 32 pixels, step of constructing displacement vectors 12 pixels, and radius of the region for vector search 8 pixels. By numerical differentiation of the vector fields, we obtained the shear strain intensity γ_i (Fig. 2a). In calculating the distributions γ_i from the model images obtained, the deviation from the values calculated analytically was $\Delta\gamma_i = 1.114 \cdot 10^{-3}$.

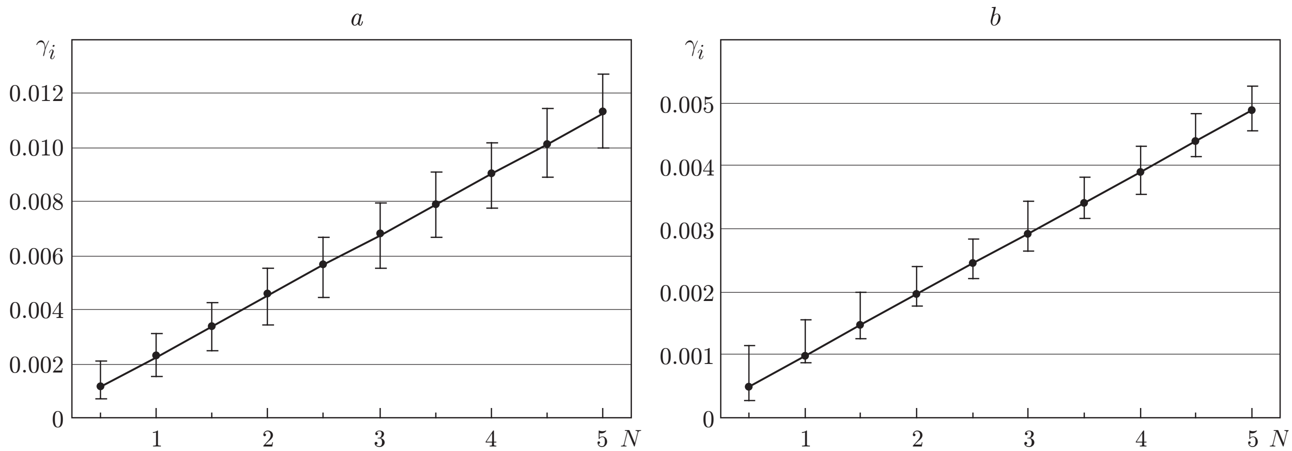

Fig. 2. Shear strain intensity calculated analytically by Eq. (3) (solid curve) and its mean arithmetic discrete distribution (N is the number of pixels): biaxial tension (a) and uniaxial shear (b).

2.2. *Model of Uniaxial Shear of the Surface.* We consider a rectangular surface area of size $l \times w$ (see Fig. 1b). In the case of a uniform shear of the surface along the x axis, the surface acquires the form of a parallelogram. The origin is located in the left bottom corner of the rectangle. The point M_1 transforms to the point M_2; the distance between the points is Δx. We write the coordinates of the radius-vectors $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}$:

$$\boldsymbol{r}_1 = (0, w), \qquad \boldsymbol{r}_2 = (\Delta x, w), \qquad \boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1 = (\Delta x, w - w) = (\Delta x, 0).$$

As the shear is uniform, the displacement vector for each point of the surface has the form

$$\boldsymbol{r}_{xy} = (ky, 0), \qquad k = \Delta x / w.$$

For all points of the surface, we can write the vector field in Cartesian coordinates:

$$\boldsymbol{V}(x, y) = V_x(x, y) \cdot \boldsymbol{i} + V_y(x, y) \cdot \boldsymbol{j}, \qquad V_x(x, y) = ky, \quad V_y(x, y) = 0.$$

Knowing the displacement parameters defined by the vector field, we can determine the longitudinal (ε_{xx}), transverse (ε_{yy}), and shear (ε_{xy}) components of the distortion tensor and the shear strain intensity γ_i:

$$\varepsilon_{xx} = \frac{\partial\,(ky)}{\partial x} = 0, \qquad \varepsilon_{yy} = 0, \qquad \varepsilon_{xy} = \frac{1}{2}\Big(\frac{\partial\,(ky)}{\partial y} + 0\Big) = \frac{k}{2}, \qquad \gamma_i = |\varepsilon_{xy}| = \Big|\frac{k}{2}\Big|.$$

Figure 2b shows the shear strain intensity calculated on the basis of the model images. The parameters of the algorithm for constructing the vector field and images were the same as those in Sec. 2.1. In calculating the mean arithmetic values γ_i from model images obtained, the difference from the analytical results was $\Delta\gamma_i = 4.72 \cdot 10^{-4}$.

3. Calculation of Strains by Analyzing Images of Surfaces of Real Objects. Some results of testing the method with the use of images of surfaces of real samples are given below. The experiments were performed with polymer samples made of polypropylene, which were loaded on an IMASh-2078 mechanical test machine under conditions of uniaxial static tension.

The experiment was performed with polypropylene samples, which had the form of a two-sided flat blade. The size of the working area of the samples was $39 \times 2 \times 1$ mm. The images were obtained in the central part of the sample, and the size of the area observed was 800×600 μm (Fig. 3a). The image size (resolution) was 768×576 pixels. The tension rate of the sample was 15 mm/h; the sample images were recorded in time intervals equal to several seconds. The field of displacement vectors constructed on the basis of two images of the sample is shown in Fig. 3b. This particular material was chosen for the experiment because it can sustain significant elastic strains, which is necessary in developing the method for strain estimates.

We describe the method for calculating the values of γ_i from the known loading rate and interval between consecutive records of the image. The tension rates of the sample area (corresponding to the image size) along the x axis and compression along the y axis are $V_x = LV_{\rm load}/L_{\rm s} = 768 \cdot 4.2/39{,}000 = 0.0827$ pixel/sec and $V_y = \mu H V_x / L = 0.5 \cdot 576 \cdot 0.0827/768 = 0.031$ pixel/sec. Here $V_{\rm load} = 15 \cdot 10^3/3600 = 4.2$ μm/sec is the loading rate

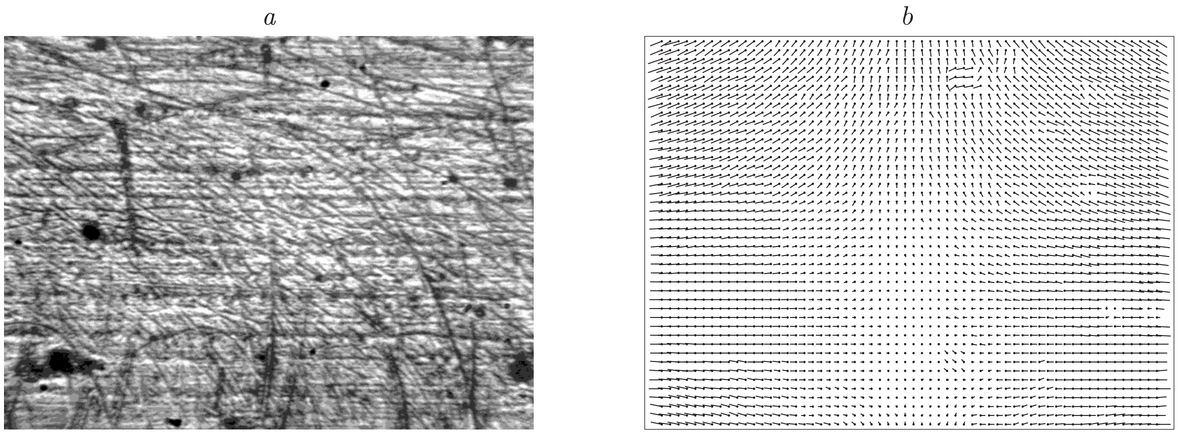

Fig. 3. Area of the polypropylene sample (a) and corresponding field of displacement vectors (b).

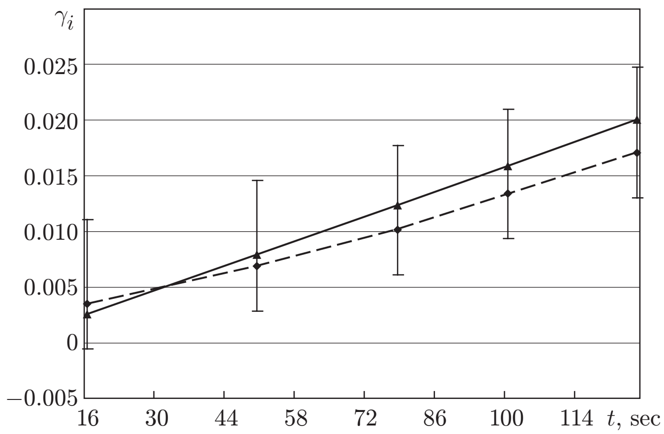

Fig. 4. Shear strain intensity in the case of tension of a polypropylene sample versus time from the beginning of recording of a series of images, obtained by the "direct" (solid curve) and "indirect" (dashed curve) calculation methods.

of the sample, $L_{\rm s}$ is the sample length, μm, L and H are the image length and width ($L \times N = 768 \times 576$ pixels), and $\mu = 0.5$ is Poisson's ratio. Knowing the interval between the image records, we can use Eq. (3) to determine the increment of strain and the components of the distortion tensor ε_{xx} and ε_{yy} for each pair of images:

$$\varepsilon_{xx} = V_x t/L, \qquad \varepsilon_{yy} = -V_y t/H.$$

Here t is the time interval between the image records; the minus sign of the component ε_{yy} corresponds to sample compression along the y axis. Knowing that $\varepsilon_{xy} = 0$ in the case of sample tension, we can calculate the shear strain intensity by Eq. (2).

Figure 4 shows the parameter γ_i as a function of time of load growth, which was calculated by the "direct" method described above and by the "indirect" method proposed in the present work and implemented in the form of a computer code. The qualitative and quantitative agreement of calculation results shows that the method proposed for strain estimates and based on an analysis of displacements of surface areas is correct and can be used together with the TOMSC optical TV measurement instrument for experimental research.

Conclusions. The method for estimating surface strains, which is based on image processing (construction of displacement vectors), is tested for the first time within the framework of methodology of physical mesomechanics.

An algorithm is proposed for determining the displacements of surface areas with subpixel accuracy, which is reached by bicubic (two-dimensional) interpolation of the greatest coefficients of the distribution of the correlation function. Better accuracy of displacement determination is reached by two-dimensional interpolation of the area rather than by consecutive interpolation by one-dimensional splines. The volume of calculations is reduced by interpolation of one necessary area and by using the search for the extreme point by means of calculating gradients instead of consecutive search in determining the maximum.

The method proposed is verified by test computations with the use of a series of model images. Model images corresponding to changes in surface shape under biaxial tension and uniaxial shear are generated. The results of the analytical and "experimental" calculations of strains are in good agreement.

The strain estimated obtained by an analytical calculation on the basis of known loading parameters and by processing of experimental images of polypropylene samples are also in good agreement. Thus, the considered method for estimating strain parameters and based on constructing the fields of displacement vectors of surface areas is correct and can be used together with the TOMSC optical TV measurement system for experimental research.

The authors are grateful to Yu. P. Stefanov for useful advice and comments on analytical methods of strain calculations, to A. V. Panin for polypropylene samples, and to I. V. Shakirov for assistance in creating model images with prescribed statistical parameters.

This work was supported by the foundation "Leading Scientific Schools of Russia" (Grant No. NSh-2324.2003.1) and by the Russian Foundation for Basic Research (Grant No. 04-01-08030-ofi-a).

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